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Discussion

Effect of friction on subsurface stresses in sliding line contact of multilayered elastic solids

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1. Introduction

Recently, Elsharkawy (1999) published the above title in the International Journal of Solid and Structures. In Sub-Section 4.2, the contact between a flat rigid punch and an elastic layer bonded or unbonded to a rigid foundation was investigated. The obtained results for the frictionless contact showed similar features to those reported by Jaffar and Savage (1988). In the case of full sliding, the effect of the coefficient of friction μ on the dimensionless contact pressure $p/p(0)$ was illustrated in Fig. 8 for a bonded layer when Poisson's ratio $\nu = 0.5$ and $a/t = 2$ (a is the contact half-width and t is the layer thickness).

The aim of this discussion is to shed more light on the above problem by considering the influences of other parameters, such as the layer compressibility and layer thickness, on the contact pressure. Also, a simple asymptotic formula for predication of the contact pressure is derived when a flat rigid punch is in full sliding contact with a bonded compressible thin layer. Finally, the full sliding contact of a rough flat punch is examined. No results will be presented for the unbonded layer problem since it is numerically similar to the bonded layer case.

2. Thin layer analysis

Bentall and Johnson (1968) showed that the normal displacement $v(x)$ due to the normal pressure $p(x)$ and the tangential traction $q(x)$ for a bonded thin layer with ($\nu \neq 0.5$) is given by

$$v(x) = \frac{(1+\nu)(1-2\nu)}{(1-\nu)E} tp(x) - \frac{(1+\nu)(1-4\nu)}{(1-\nu)E} \frac{t^2}{2} \frac{\partial q(x)}{\partial x}. \quad (1)$$

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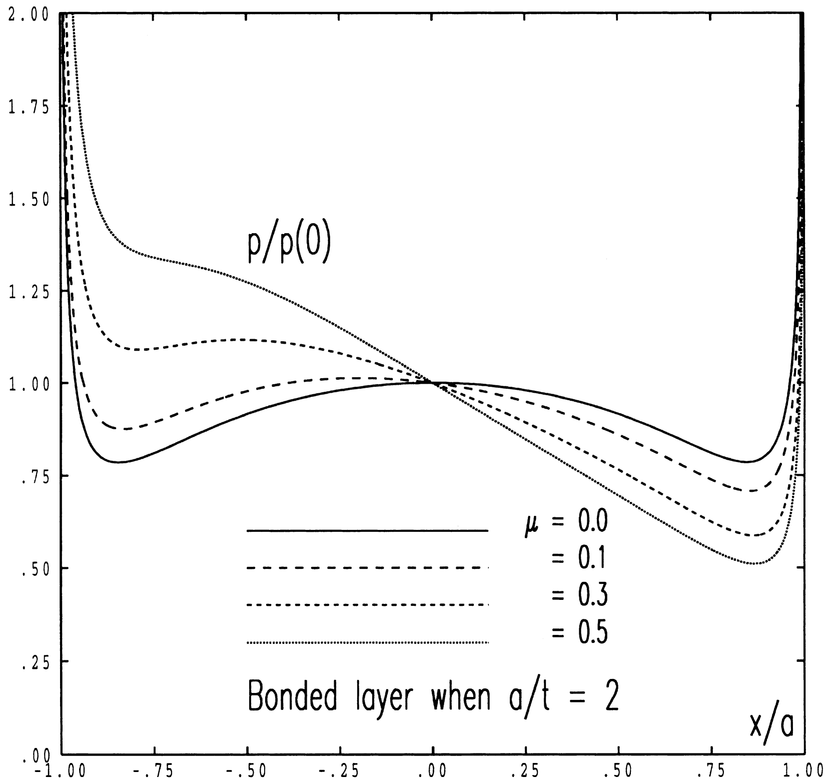

Fig. 1. Dimensionless contact pressure for a smooth flat punch.

It is clear from Eq. (1) that $q(x)$ makes no contribution when $\nu = 0.25$ and the contact pressure becomes symmetric. Assuming that $q(x) = \mu\, p(x)$ and $v(x) = v^*$, then Eq. (1) has the solution

$$p(x) = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \frac{v^*}{t} + C\exp(Ax/t), \tag{2}$$

where C is a constant of integration and $A = [2(1-2\nu)]/[(1-4\nu)\mu]$. If $p(0)$ is the contact pressure at $x = 0$, then Eq. (2) becomes

$$p(x) - p(0)\exp(Ax/t) = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \frac{v^*}{t}(1 - \exp(Ax/t)). \tag{3}$$

It is worth remarking that when $\mu > 0$ the value of A changes sign, i.e. $A > 0$ when $\nu < 0.25$ and vice versa. This means that the position of the minimum contact pressure moves from one side of $x = 0$ to the other side. Close inspection of Eq. (2) reveals that the contact pressure is finite at the contact ends rather than infinite as one expects. Hence, Eq. (2) is valid for most of the contact area, although it is not valid in the neighbourhood of the contact ends.

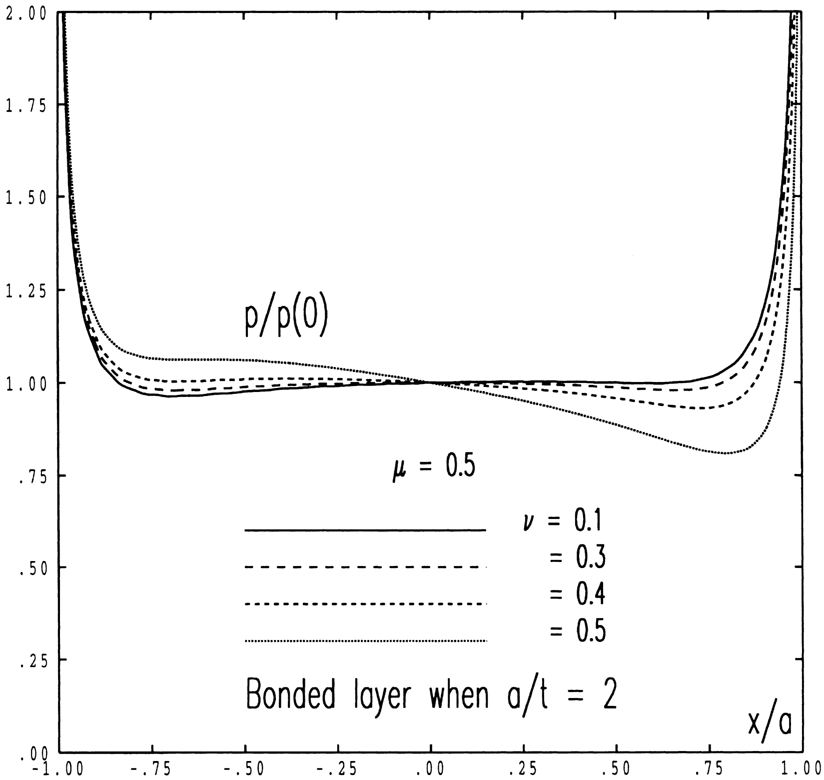

Fig. 2. Influence of layer compressibility on contact pressure.

3. Results

Jaffar (1991) extended the numerical method developed in Jaffar and Savage (1988) in order to investigate the full sliding in layered contact. The cylindrical punch was only examined because the numerical results can be checked with the existing solutions. It was shown that the obtained data agreed well with the solutions of a half-plane (Johnson, 1985) and a thin unbonded layer (Conway, 1971). Since the method can be applied to any punch profile of a polynomial of degree n, Fig. 8 of Elsharkawy (1999) was produced using the method described in Jaffar (1991) and shown in Fig. 1. Despite it being impossible to compare both data, a close examination reveals that both results showed similar features (e.g. the minimum value of $p/p(0)$ is around 0.5 when $\mu = 0.5$).

Elsharkawy wrote "It is apparent that the influence of frictional traction upon normal pressure for this case is significant". His remark was observed in Jaffar (1991) and does not take into account the effect of the layer compressibility and layer thickness. The influence of the layer compressibility on $p/p(0)$ is exhibited in Fig. 2 for a bonded layer when $a/t = 2$ and $\mu = 0.5$. It is interesting to see that the location of the minimum value of $p/p(0)$ occurs on the entry side for $\nu = 0.1$ and on the exit side for $\nu = 0.3$, 0.4 and 0.5. This is consistent with discussion of Eq. (2) in the previous section. The variation of $p/p(0)$ with the layer thickness is shown in Fig. 3 when $\mu = 0.3$ and $\nu = 0.4$. As the layer thickness is reduced the contact pressure becomes flat over most of the contact area. On the other hand, a pressure dip evolved (as with frictionless contact) which is more pronounced on the exit side of the contact.

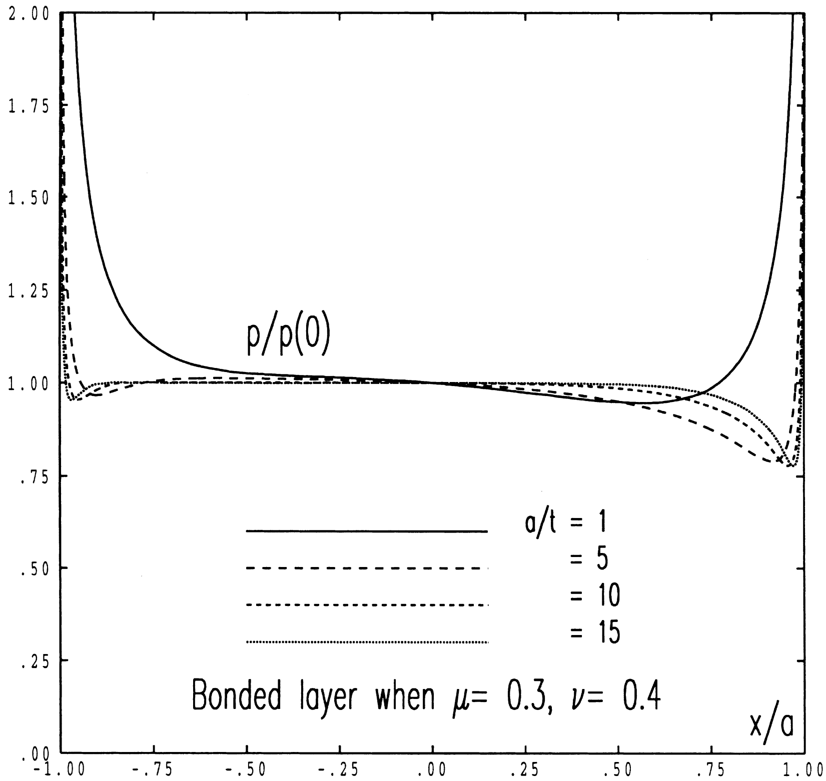

Fig. 3. Influence of layer thickness on contact pressure.

4. Rough flat punch

Johnson (1985) proposed a one-dimensional model for a regular wavy surface whose profile was described by a sinusoidal function. Jaffar (1997) has modified the method developed in Jaffar and Savage (1988) in order to study the frictionless layered contact using Johnson's model. Both rough flat and rough cylindrical punches were examined when the layer is either bonded or unbonded. Moreover, a set of asymptotic solutions for the contact pressure was presented for compressible and incompressible thin layers.

The non-dimensional profile $V(X)$ of a flat punch having a sinusoidal roughness is given by

$$V(X) = V^* - B\left(1 - \cos\frac{2\pi X}{\Lambda}\right), \tag{4}$$

where $B = b/a$, $V(X) = (v(x))/a$ and $V^* = v^*/a$. b is the amplitude and λ is the wavelength.

For rough contact, we only consider the case examined in Fig. 1. The reason for this is because sinusoidal roughness shows similar features to the corresponding smooth contact. The only difference is a regular wavy contact pressure instead of a smooth pressure. Dimensionless pressure distributions for a

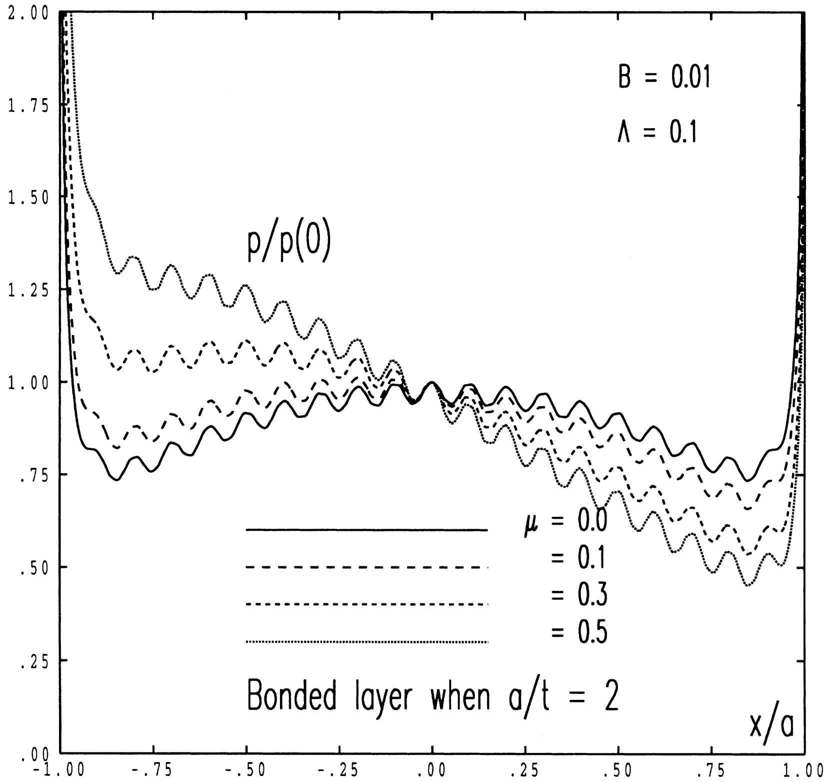

Fig. 4. Dimensionless contact pressure for a rough flat punch.

rough flat punch are plotted in Fig. 4 when the amplitude parameter $B = 0.01$ and the wavelength $\Lambda = 0.1$.

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